Factor Alignment for Equity Portfolio Management

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Factor Alignment Basics
A Decomposition of Expected Returns

\[
\max_h \alpha^T h - \frac{1}{2} h^T Q h
\]

\[
Q = X\Omega X^T + \Delta
\]

\[
\alpha = \alpha_{\perp} + \alpha_X
\]

The portion of alpha explained by the risk factors is referred to as the spanned component.

If alpha and risk factors are aligned, then \( \alpha_{\perp} = 0 \), or, in other words, there is misalignment if and only if \( \alpha_{\perp} \neq 0 \).

The residual obtained by regressing the alphas against the factors in the risk model is referred to as the orthogonal component of alpha.
Why is Misalignment “Bad” in MVO?

In MVO, we are aiming to create portfolios that have an optimal risk-adjusted expected return.

- If a portion of systematic risk is not accounted for then the resulting risk-adjusted expected return cannot be “optimal”.

The optimizer sees no systematic risk in the orthogonal component of alpha and is hence likely to load up on it.

\[
\alpha = \alpha_{\perp} + \alpha_{X}
\]

- No Factor Risk, Only Specific Risk
- Contains Factor Risk and Specific Risk

\[
\max_h \alpha^T h - \frac{1}{2} h^T Qh
\]
Alignment is Also About Constraints

Implied Alpha

\[
\max_h \alpha^T h - \frac{\lambda}{2} h^T Qh \\
\text{s.t.} \quad \text{Constraint 1} \\
\quad \vdots \\
\quad \text{Constraint } m
\]

\[
\max_h \alpha^{*T} h - \frac{\lambda}{2} h^T Qh
\]

- Implied alpha acts as the *de facto* alpha in the case of constrained MVO problems
- Optimizer sees no systematic risk in the orthogonal component of implied alpha and is hence likely to load up on it
- Implied alpha is a *dynamic* entity determined by the interaction of alpha, risk factors and constraints
Alpha vs Implied Alpha Misalignment

- Risk Ellipse
- Implied Alpha
- Alpha
- Optimal Portfolio With constraint
- No longer feasible!
Misalignment Problems and Two Ways Out

Problem:
- Misalignment due to proprietary factors not being represented in the risk model – Having non-zero orthogonal components

Solution:
- **Custom Risk Models** – Add the proprietary factors to the risk models and completely regenerate them

Problem:
- Misalignment due to the usage of constraints – The difference between alpha and implied alpha (even with Custom Risk Models)

Solution:
- **Alpha Alignment Factor Methodology** – Add to the risk model the orthogonal component of implied alpha (Axioma proprietary and patented)

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From Misalignment To Alignment

Base Model

Custom Risk Model

Base Model + AAF

Custom Risk Model + AAF
How To Align Proprietary Factors With Custom Risk Models

Base Model → Custom Risk Model

Base Model + AAF → Custom Risk Model + AAF

Alpha Misalignment

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How To Align Constraints

Base Model

Alpha + Constraint
Misalignment (approx)

Base Model + AAF

Custom Risk Model

Custom Risk Model + AAF
How To Align Constraints AND Proprietary Factors

Base Model

Base Model + AAF

Alpha Misalignment

Custom Risk Model

Custom Risk Model + AAF

Alpha + Constraint Misalignment (approx)

Constraint Misalignment (approx)
Why Do We Have Misalignment,
And The Case For And Against
Alignment
Independent Alpha, Risk and Construction Processes Generate Misalignment

- Alpha Process
- Risk Process
- Portfolio Construction Strategy
- Optimizer
- Optimal Portfolio

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To Align or Not To Align? (Proprietary Factors & Constraints)

Proprietary Factors: Against

• The **Free Lunch Theory**: “I don’t want my factors in the risk model, otherwise the risk model will not let me bet on them” – (Never mind the systematic risk)

Constraints: Indifferent

• The presence of constraints is ignored in most of the literature that concerns alignment issues
Empirical Evidence of Why Alignment Matters

The USER Model* and Client Data

* Guerard et. al
Proprietary Factors in the USER Model have Orthogonal Components with Realized Systematic Risk Comparable with Other Axioma Factors
Proprietary Factors Have Statistically Significant Orthogonal Components

Percentage of statistically significant periods (90% cf)

Axioma Style Factors, 25-75% Range % of SSP
Average Correlation Between Alpha and Implied Alpha Is Low for Most Clients

Correlation

Fundamental Model
Statistical Model

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Correlation between Alpha and Implied Alpha Changes Significantly Over Time, Even For Perfectly Aligned Risk Models
The Opportunity Cost of Misalignment: Experiments with the USER Model
A Practical Active Strategy: USER Model

Maximize Expected Return

s.t.

• Fully invested long only portfolio
• GICS Sector exposure constraints (20%)
• GICS Industry exposure constraints (10%)
• Active asset bounds constraint (2%)
• Turnover Constraint (16% two-way)
• Active Risk Constraint (3%)

Base Model = US2AxiomaMH (Axioma Fundamental Model)
Benchmark = Russell 3000
Monthly backtest, 1999-2009 time period
Expected Return = USER.BP + US2AxiomaMH.Medium-Term-Momentum

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Predicted vs Realized Active Risk
Significantly Improves with CRM + AAF

- Risk Target: 3.00%
- Base Model: 3.81%
- CRM: 3.38%
- CRM + AAF: 3.08%

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The Realized Risk-Return Frontier Moves Upwards (Annualized Active Returns and Risk)

Frontier Spreads:
CRM: 1% Additional Realized Active Annualized Return
CRM + AAF: 1.75% Additional Realized Active Annualized Return
Frontier Spreads Should be Interpreted as Opportunity Costs for Misalignment

Opportunity Cost of misalignment from constraints
Relaxing Asset Bounds Further Improves the Frontier Spreads
Relaxing the Industry Exposure Bounds Also Improves the Frontier Spreads
Increasing the Turnover Limit Improves Frontier Spreads
Turnover Utilization Improves with CRM + AAF (Portfolios with Similar Realized Active Risk)
The Opportunity Cost of Misalignment:
Experiments with Client Data*
Maximize Expected Return – Active Variance

s.t.

• 130/30 long short portfolio
• GICS Sector exposure constraints (20%)
• GICS Industry exposure constraints (10%)
• Active asset bounds constraint (2%)
• Turnover Constraint (30% two-way)
• Maximum shorting constraint

Base Model = US2AxiomaMH (Axioma Fundamental Model)
Benchmark = Russell 1000
Monthly backtest, 2002-2011 time period
Expected Return = Dynamically varying combination of proprietary factors

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Improvements From Alignment Are Also Significant for More Complex Alpha Models

![Graph showing realized active return vs realized active risk with three lines representing US2AxiomaMH, CRM, and CRM+AAF.]
When are FAP solutions most valuable?
The Outperformance of CRM+AAF Is Very Strongly Correlated With the Latent Volatility of the Orthogonal Component of Implied Alpha
The Performance Differential Using AAF in Different Market Regimes Has 60% Correlation With Latent Volatility of the Orthogonal Part of Implied Alpha.
Theoretical Foundations: Pushing Frontier Theorem (Saxena and Stubbs)

Theorem

\[ U(Q_y) = U(Q) + \frac{1}{2\sigma^4} \left( \frac{Systematic\ Risk(y)}{Total\ Risk(y)} \right)^2 \]

- The increment in the utility function that results when FAP solutions such as AAF or CRM are employed increases as a function of systematic risk associated with hidden systematic risk factors.
- Periods of high cross sectional correlations are often accompanied by rising factor volatilities of both common and hidden systematic risk factors (Renshaw and Saxena, 2011).
- The incremental value of FAP solutions tends to be highest during periods of high cross sectional correlations as we are currently witnessing.

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Lessons Learned From
Client Implementations
Include individual components of alpha as distinct custom factors

It increases the flexibility of the optimizer in finding better risk/return trade-offs (example: a medium-return, high-volatility component combined with a low-volatility, medium-return component)
Generate frontiers to analyze risk-adjusted performance

Portfolios with similar levels of realized risk should be compared

If a risk constraint rarely binds, addressing mis-alignment will have limited impact on portfolio construction
Re-examine your strategy and consider loosening constraints

Many constraints are typically used to compensate for risk under-prediction in traditional MVO. CRMs provide greatly improved risk estimates, which may obviate the need for tight constraints.
Observations and Conclusions

Q1. *What are the sources of factor alignment problems (FAP)?*

A. **Independent alpha, risk, and strategy design processes**

Q2. *What is the opportunity cost of FAP?*

A. **Pushing realized frontier upwards**

Q3. *When are FAP solutions most valuable?*

A. **During periods of high cross sectional correlations**
References


• A. Saxena and R. A. Stubbs, Alpha alignment factor: A solution to the underestimation of risk for optimized active portfolios. *Journal of Risk*, To Appear.


