Robust Portfolio Control with Stochastic Factor Dynamics

Paul Glasserman and Xingbo Xu
Columbia University

19th Annual Financial Engineering Conference
Center for Financial Engineering
Columbia University

November 9, 2012
Summary

• We develop a portfolio control strategy with the following features
  – “Alpha” factor dynamics/return predictability
  – Transactions costs
  – Tractability with a relatively large number of assets
  – Robust to model error in the specification of the stochastic dynamics of the model

• Backtesting shows that robustness is valuable in out-of-sample tests with the model estimated on a rolling window of data

• By acknowledging model error, the robust strategy bets less aggressively on signals from the factors and avoids the worst losses
Outline

• Robustness and why it matters for portfolio selection

• Portfolio control with stochastic “alpha” factors

• Robust portfolio control

• Case study in commodity futures
Background on Robustness
Model Error + Optimization = Dangerous Combination

• Suppose you have a model of risk and return
  – If you’re overestimating and underestimating equally often, you’re doing well

• Now optimize. Optimization will drive you to invest in assets for which you have
  – Overestimated the expected return
  – Underestimated the risk

• Optimization brings out the worst in models, amplifying the impact of errors

• Robustness seeks to guard against this by acknowledging model error from the outset
Portfolio Optimization: Warm-Up

- Consider a classical mean-variance portfolio optimization problem

\[
\max_{x_1, \ldots, x_d} \sum_{i=1}^{d} x_i \mu_i - \frac{\gamma}{2} \sum_{i, j=1}^{d} x_i x_j \sigma_i \sigma_j \rho_{ij}
\]

or, in vector notation

\[
\max_{x \in \mathbb{R}^d} x^\top \mu - \frac{\gamma}{2} x^\top \Sigma x
\]

where

\[x = \text{vector of portfolio positions}\]
\[\mu = \text{vector of mean returns}\]
\[\Sigma = \text{covariance matrix of returns}\]
\[\gamma = \text{risk-aversion parameter}\]
Robust Formulation

• A robust optimization formulation:

$$\max_{x \in \mathbb{R}^d} \min_{\mu \in \mathcal{M}, \Sigma \in \mathcal{S}} x^\top \mu - \frac{\gamma}{2} x^\top \Sigma x$$

• Here, $\mathcal{M}$ and $\mathcal{S}$ are “uncertainty sets” in which the true mean and covariance may lie – acknowledges model uncertainty

• The goal is to optimize the portfolio against a worst-case configuration of parameters chosen by an adversary

• Ben-Tal et al. (2000), Goldfarb-Iyengar (2003), Bertsimas and Pachamanova (2008), and many others
From Parametric Robustness to Stochastic Robustness

• The formulation

\[
\max_{x \in \mathbb{R}^d} \min_{\mu \in \mathcal{M}, \Sigma \in \mathcal{S}} \ x^\top \mu - \frac{\gamma}{2} x^\top \Sigma x
\]

limits uncertainty (model error) to parameters

• This is adequate for a static (one-period) problem

• For *dynamic* portfolio control, we are concerned about model error in
  – the evolution of market factors
  – the “alpha” model of excess returns

• We want robustness to errors in the *stochastic dynamics* of the model, not just parameters
Dynamic Portfolio Control: Non-Robust Version
Underlying Model: Garleanu-Pedersen (2011)

- Factor model for returns

\[ r_{t+1} = \mu + B f_t + u_{t+1}, \]

- Factors could be, e.g., momentum factors calculated from past returns
- Factors introduce some predictability in returns – signals on which to trade
Factor Dynamics

• Signals don’t last forever:

\[ f_{t+1} = C f_t + v_{t+1}, \]

Mean-reversion matrix \( N(0, \Sigma_v) \)

• Factors mean-revert to zero (and randomly jump)

• Some factors may persist longer than others – different speeds of mean-reversion
Performance Objective - 1

Portfolio $x_t = \text{number of shares of each asset}$

Objective: Each period, risk-adjusted excess return

$$x_t^\top (r_{t+1} - \mu) - \frac{\gamma}{2} x_t^\top \Sigma u x_t$$
Performance Objective - II

Portfolio $x_t = \text{number of shares of each asset}$

Objective: Net of transaction costs

$$x_t^\top (r_{t+1} - \mu) - \frac{\gamma}{2} x_t^\top \Sigma u x_t - \frac{1}{2} \Delta x_t^\top \Lambda \Delta x_t$$

If price impact is proportional to trade size $\Delta x$ for small trades, then cost is approximately quadratic in trade size.
Performance Objective - III

Portfolio $x_t = \text{number of shares of each asset}$

Objective: Discounted and summed

$$E \left[ \sum_{t=0}^{\infty} \beta^t \left( x_t^T (r_{t+1} - \mu) - \frac{\gamma}{2} x_t^T \Sigma u x_t - \frac{1}{2} \Delta x_t^T \Lambda \Delta x_t \right) \right]$$
Performance Objective - IV

Portfolio $x_t = \text{number of shares of each asset}$

Objective: Equivalently, given factor model,

$$E \left[ \sum_{t=0}^{\infty} \beta^t \left( x_t^\top (B f_t + u_{t+1}) - \frac{\gamma}{2} x_t^\top \Sigma_u x_t - \frac{1}{2} \Delta x_t^\top \Lambda \Delta x_t \right) \right]$$

Because

$$r_{t+1} = \mu + B f_t + u_{t+1}$$

Investor optimizes over trading strategy $\Delta x$
Solution

- This is a linear-quadratic control problem, solution given by Garleanu-Pedersen (2011)

- Optimal objective function is quadratic in the state \((x_{t-1}, f_t)\)

- Optimal trade is linear in the state
Dynamic Portfolio Control: Adding Robustness
How Might The Model Be Wrong?

• We might have the wrong factor loadings for returns
  \[ r_{t+1} = \mu + B \mathbf{f}_t + u_{t+1}; \]

• We might have the wrong speed of mean reversion for the factors
  \[ \mathbf{f}_{t+1} = C \mathbf{f}_t + v_{t+1}; \]

• We might have the wrong long-run level for the factors

• Errors may not be Gaussian

• Errors may be correlated

• Etc.
Robustness

• We would like to focus on the most *likely* most *harmful* error

• Most likely
  – Suppose we’ve estimated the model from data
  – Other models may be consistent with the data, to varying degrees
  – *Relative entropy* is a measure of proximity of an alternative model to the baseline model – Hansen-Sargent (2007) robustness framework

• Most harmful
  – Consider all alternatives within a given relative entropy “distance”
  – Optimize against the worst one
  – Think of a malicious adversary who changes the market dynamics as you trade, subject to a relative entropy budget
Robust Control Problem

Investor optimizes against the worst-case model error

The adversary is penalized for changing the model too much

The parameter $\theta$ controls the degree of pessimism or potential model error
Solution

- Extends non-robust case

- Optimal objective function is again quadratic in the state \( (x_{t-1}, f_t) \)

- Optimal control is again linear in the state

- As easy to solve as the non-robust version (which becomes a special case)
Case Study in Commodity Futures
Data

- Daily, Jan 1, 1996 – April 9, 2010
- 15 commodity futures

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>aluminum, copper, nickel, zinc, lead, tin</td>
<td>LME</td>
</tr>
<tr>
<td>gas oil</td>
<td>Intercontinental</td>
</tr>
<tr>
<td>WTI, unleaded gasoline, natural gas</td>
<td>NYMEX</td>
</tr>
<tr>
<td>gold, silver</td>
<td>COMEX</td>
</tr>
<tr>
<td>coffee, cocoa, sugar</td>
<td>NYBOT</td>
</tr>
</tbody>
</table>

- Excludes commodities with tight price limits
- Price change time series constructed from rolls to liquid contracts
Factor Model

- 3 factors per commodity: 5-day, 1-year, and 5-year moving averages
- Pooled panel regression estimated with weighted least squares

\[ r_{t+1}^s = 0.004 + 11.43 f_{t}^{5D,s} + 107.55 f_{t}^{1Y,s} - 218.76 f_{t}^{5Y,s} + u_{t+1}^s, \]

\[ \Delta f_{t+1}^{5D,s} = -0.2510 \Delta f_{t}^{5D,s} + v_{t+1}^{5D,s}, \]
\[ \Delta f_{t+1}^{1Y,s} = -0.0039 \Delta f_{t}^{1Y,s} + v_{t+1}^{1Y,s}, \]
\[ \Delta f_{t+1}^{5Y,s} = -0.0010 \Delta f_{t}^{5Y,s} + v_{t+1}^{5Y,s}. \]

- Quite close to estimates in Garleanu-Pedersen
Evaluating Performance

We evaluate

- In-sample performance: model and results estimated from full data
- Out-of-sample performance: model estimated on a rolling window

- Objective function to which control is optimized
  - Sharpe ratio

- Robustness to errors in return model and factor dynamics
- Robustness to errors in factor dynamics only

- Different levels of robustness parameter $\theta$
- Comparison with various simple scaling heuristics
### In-Sample Tests

<table>
<thead>
<tr>
<th>θ</th>
<th>Gross</th>
<th>t-stat</th>
<th>Net</th>
<th>t-stat</th>
<th>Gross</th>
<th>t-stat</th>
<th>Net</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>No TC</td>
<td>0.82</td>
<td>-11.67</td>
<td>-1.22</td>
<td>-159.68</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Myopic</td>
<td>0.08</td>
<td>0.56</td>
<td>0.08</td>
<td>0.70</td>
<td>0.62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-robust</td>
<td>0.60</td>
<td>0.70</td>
<td>0.82</td>
<td>1.48</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Robust</td>
<td>10^{10}</td>
<td>0.61</td>
<td>0.57</td>
<td>0.35</td>
<td>0.70</td>
<td>-0.01</td>
<td>0.62</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>10^{9}</td>
<td>0.61</td>
<td>0.57</td>
<td>0.10</td>
<td>0.55</td>
<td>-0.61</td>
<td>0.51</td>
<td>-0.45</td>
</tr>
<tr>
<td></td>
<td>10^{8}</td>
<td>0.63</td>
<td>0.57</td>
<td>0.04</td>
<td>0.15</td>
<td>-1.20</td>
<td>0.13</td>
<td>-1.05</td>
</tr>
<tr>
<td></td>
<td>10^{7}</td>
<td>0.72</td>
<td>0.66</td>
<td>0.65</td>
<td>0.00</td>
<td>-1.33</td>
<td>0.00</td>
<td>-1.17</td>
</tr>
<tr>
<td></td>
<td>10^{6}</td>
<td>0.79</td>
<td>0.77</td>
<td>1.23</td>
<td>0.00</td>
<td>-1.36</td>
<td>0.00</td>
<td>-1.19</td>
</tr>
<tr>
<td></td>
<td>10^{5}</td>
<td>0.82</td>
<td>0.82</td>
<td>1.48</td>
<td>0.00</td>
<td>-1.36</td>
<td>0.00</td>
<td>-1.20</td>
</tr>
</tbody>
</table>

- Dynamic control rules are much better than naïve strategies
- As expected, robustness does not improve the objective to which the original control is optimized
- Robustness may improve the Sharpe ratio, but the difference does not appear to significant
- Standard errors for differences estimated by batching data
Out-of-Sample Tests

- Re-estimate model every week using previous 6 months of data
- Follow investment out-of-sample for the subsequent week

- Robustness improves performance by all measures
- Results are not very sensitive to robustness penalty $\theta$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Performance Ratio</th>
<th>Objective $\times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gross</td>
<td>$t$-stat</td>
</tr>
<tr>
<td>No TC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Myopic</td>
<td>0.53</td>
<td>-5.74</td>
</tr>
<tr>
<td>non-robust</td>
<td>-0.57</td>
<td>-0.59</td>
</tr>
<tr>
<td>robust</td>
<td>0.35</td>
<td>0.04</td>
</tr>
<tr>
<td>$10^9$</td>
<td>0.39</td>
<td>0.95</td>
</tr>
<tr>
<td>$10^8$</td>
<td>0.40</td>
<td>0.46</td>
</tr>
<tr>
<td>$10^7$</td>
<td>0.46</td>
<td>0.68</td>
</tr>
<tr>
<td>$10^6$</td>
<td>0.52</td>
<td>0.90</td>
</tr>
<tr>
<td>$10^5$</td>
<td>0.53</td>
<td>0.90</td>
</tr>
<tr>
<td>$10^4$</td>
<td>0.53</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Robust portfolio is usually – but not always – less aggressive

Myopic strategy lags because it fails to predict factor moves
Out-of-Sample Tests: Net Returns

• Much of the benefit of robustness occurs on a small number of extreme days
• Dynamic controls much better than myopic strategy
Out-of-Sample Tests: Net Returns

European central banks sign agreement on gold reserves

Big bet on sugar
Near September 27, 1999
Prices Near February 6, 2006

The four largest positions at that time are in these commodities.
Positions in gold and zinc are similar across the two strategies, but robustness avoids the large bet on sugar.
Summary

• We have developed a portfolio control strategy with the following features
  – Robustness to model error in the specification of the stochastic dynamics of the model
  – “Alpha” factor dynamics/return predictability
  – Transactions costs
  – Tractable for a reasonably large number of assets

• Backtesting shows that robustness is valuable in out-of-sample tests with model estimated on a rolling window of data

• By acknowledging model error, the robust strategy bets less aggressively on signals from the factors and avoids the worst losses

• Paper available on SSRN
Thank You