Attilio Meucci

Fully Integrated Liquidity- and Market-Risk Model


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Conceptual issues in liquidity modeling
Conceptual issues in liquidity modeling

- **paths of market risk drivers**
- **portfolio value in scenario 1**
- **portfolio value in scenario 2**
- **portfolio value in scenario 3**
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Conceptual issues in liquidity modeling

profit

loss

P&L distribution

Tail risk
Conceptual issues in liquidity modeling

Value in scenario 1 unknown due to liquidity

Value in scenario 2 unknown due to liquidity

Value in scenario 3 unknown due to liquidity

Profit vs. Loss diagram showing fluctuations in each scenario.
Conceptual issues in liquidity modeling

Profit

Loss

Liquid securities display
Small price uncertainty
illiquid securities display large price uncertainty
liquidity hits only in undesirable direction
liquidity hits harder when we are in trouble
Conceptual issues in liquidity modeling

- profit
- loss

liquidity risk
- market risk
- “+”
- ?
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Market risk as Fully Flexible Probabilities (FFP)

\[ X \sim \{ x_j, p_j \}_{j=1, \ldots, J} \]

Paths of market risk drivers (interest rates, stock returns, implied volatility surfaces, ...)

probability of paths

\[ X_j \]

joint scenario of N risk drivers

probability of joint scenario

\[ J \]

\[ N \]
\[ \mathbf{X} \sim \{ \mathbf{x}_j, p_j \}_{j=1,\ldots,J} \]

Paths of market risk drivers (interest rates, stock returns, implied volatility surfaces, ...)

\[ \Pi_n = \pi_n (\mathbf{X}) \]

\[ \sim \{ \pi_n (\mathbf{x}_j), p_j \}_{j=1,\ldots,J} \]

Market P&L of each security
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Liquidity-, Funding- and Market-Risk Paths of market risk drivers (interest rates, stock returns, implied volatility surfaces,...)

\[ X \sim \{ x_j, p_j \}_{j=1,...,J} \]

Paths of market risk drivers (interest rates, stock returns, implied volatility surfaces,...)

\[ \Pi_n = \bar{\pi}_n (X) \]
\[ \sim \{ \bar{\pi}_n (x_j), p_j \}_{j=1,...,J} \]

Market P&L of each security

\[ \Pi \equiv \sum_n h_n \Pi_n \]
\[ \sim \{ \sum_n h_n \bar{\pi}_n (x_j), p_j \}_{j=1,...,J} \]

Market P&L of portfolio

Market risk as Fully Flexible Probabilities (FFP)
Liquidity adjustment depends on
- market conditions
- liquidation policy

Market-dependent corrections (VIX, liquidity indices,…)

\[
\Delta \Pi_n \sim N \left( \mu_n, \sigma_n^2 \right)
\]

\[
\begin{align*}
\mu_n &= -\alpha_n(\mathbf{x}) e_n |\Delta h_n| - \beta_n(\mathbf{x}) e_n \bar{\sigma}_n \frac{|\Delta h_n|^{3/2}}{\sqrt{v_n}} \\
\sigma_n &= \delta_n(\mathbf{x}) \sqrt{v_n} e_n |\Delta h_n|
\end{align*}
\]

Liquidity adjustment of portfolio

\[
\Delta \Pi \equiv \sum_n \Delta \Pi_n \sim N \left( \mu, \sigma^2 \right)
\]

\[
\begin{align*}
\mu &= \sum_n \mu_n \\
\sigma^2 &= \sum_{n,m} \sigma_n \sigma_m \rho_{n,m}
\end{align*}
\]
\[ \Pi = \Pi_\text{old} + \Delta \Pi \]

portfolio total P&L

market risk P&L  
liquidity risk adjustment

Total risk = market + liquidity
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\[ \Pi = \overline{\Pi} - \Delta \Pi \]

portfolio total P&L

market risk P&L
liquidity risk adjustment

Total P&L distribution = mixture of bells with different centers and widths

\[ f_{\Pi}(z) = \sum_{j} p_{j} \varphi \left( \frac{z - \overline{\pi}_{j} - \mu_{j}}{\sigma_{j}} \right) \]

Total risk = market + liquidity
\[ \Pi = \overline{\Pi} + \Delta\Pi \]

portfolio total P&L

\[ \uparrow \quad \uparrow \]

market risk

liquidity risk

risk P&L

adjustment

Total P&L distribution = mixture of bells with different centers and widths

\[ f_\Pi (z) = \sum_j \frac{p_j}{\sigma_j} \varphi \left( \frac{z - \overline{\pi}_j - \mu_j}{\sigma_j} \right) \]

Illiquidity score

\[ Ill = \left| CVaR \left\{ \overline{\Pi} \right\} - CVaR \left\{ \Pi \right\} \right| \]

\[ \frac{CVaR \left\{ \overline{\Pi} \right\}}{CVaR \left\{ \Pi \right\}} \]
\[ \Pi = \Pi + \Delta \Pi \]  
portfolio total P&L

\[ \uparrow \quad \left\uparrow \right\uparrow \]  
market risk P&L  
liquidity risk adjustment

Marginal contributions

\[ CVaR\{\Pi\} = \partial_{\Pi} CVaR\{\Pi\} + \partial_{\Delta \Pi} CVaR\{\Pi\} \]  
market risk  
liquidity risk
\[ \Pi = \Pi_0 + \Delta \Pi \]

**portfolio total P&L**

\[ \uparrow \quad \uparrow \]

**market risk**  **liquidity risk**

**risk P&L**  **adjustment**

**Market-Liquidity risk decomposition**

**Marginal contributions**

\[ CVaR \{ \Pi \} = \partial_\Pi CVaR \{ \Pi \} + \partial_{\Delta \Pi} CVaR \{ \Pi \} \]

\[ \underbrace{\text{market risk}}_{\partial_\Pi CVaR \{ \Pi \}} + \underbrace{\text{liquidity risk}}_{\partial_{\Delta \Pi} CVaR \{ \Pi \}} \]

\[ \partial_{\Delta \Pi} CVaR \{ \Pi \} = \sum_j p(j) \frac{\mu(j) - \sigma(j) \varphi(z(j))}{\Phi(z(j))} \]

\[ z(j) \equiv (F^{-1}_\Xi(\alpha) - \bar{\pi}(j) - \mu(j))/\sigma(j) \]
Figure 2: Liquidity risk has one-sided, adverse impact on the portfolio.
Figure 3: Market risk is more diversifiable than liquidity risk.
Figure 4: Funding risk: state-dependent liquidation schedules
Figure 5: Fully Flexible Probabilities framework addresses estimation risk with no additional computational cost

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