

Abstract

Optimizing a multi-objective portfolio is a challenging task given that performance measures are represented in different magnitudes. A common practice becomes to optimize a single measure while adding other measures as constraints. The exact constraint depends on the user risk appetite, but it can also be a challenge given that the overall level of the measure will vastly differ depending on market conditions.

Explainability Index

In our previous paper (1), we propose an easily explainable measure (Explainability Index) that allows to combine as many measures as desired while accounting for their magnitude differences and market conditions. While previous work focused on calculating Explainability Index for a given asset, we now focus on obtaining an allocation/portfolio with the lowest possible EI given expectations.

Constructing an allocation based on EI

Algorithm 1 EI Allocation Algorithm

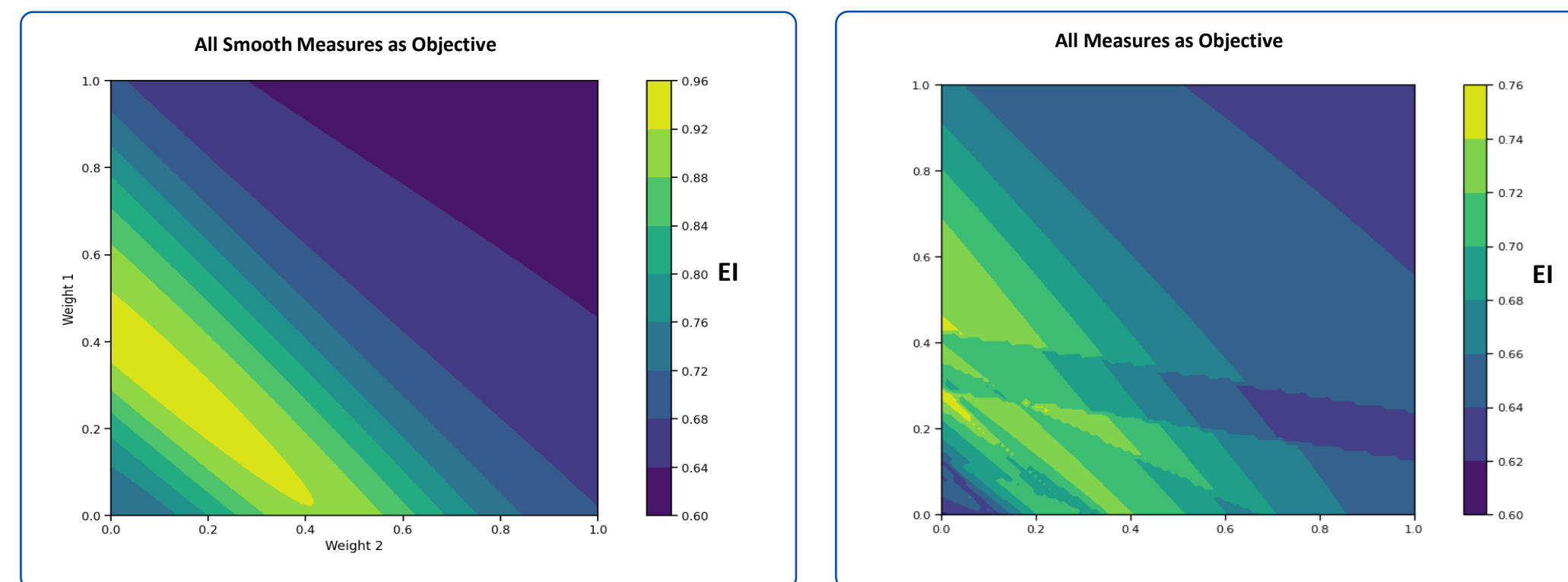
- 1: Grab the past N monthly returns for each asset in the portfolio
- 2: Calculate the cumulative return of each asset
- 3: Multiply by the portfolio weights X
- 4: Convert cumulative portfolio return to monthly returns
- 5: Calculate performance measures based on the previous monthly returns
- 6: Calculate EI
- 7: Optimize X such that EI is minimized

- The optimization method is not quadratic and it will required the application of gradient descend
- For the initial guess, we solve the closest possible single objective optimization via traditional allocation methods, e.g. MVO, NCO, HRP, HERC.
- We pick the initialization that yields the lowest EI

Challenges

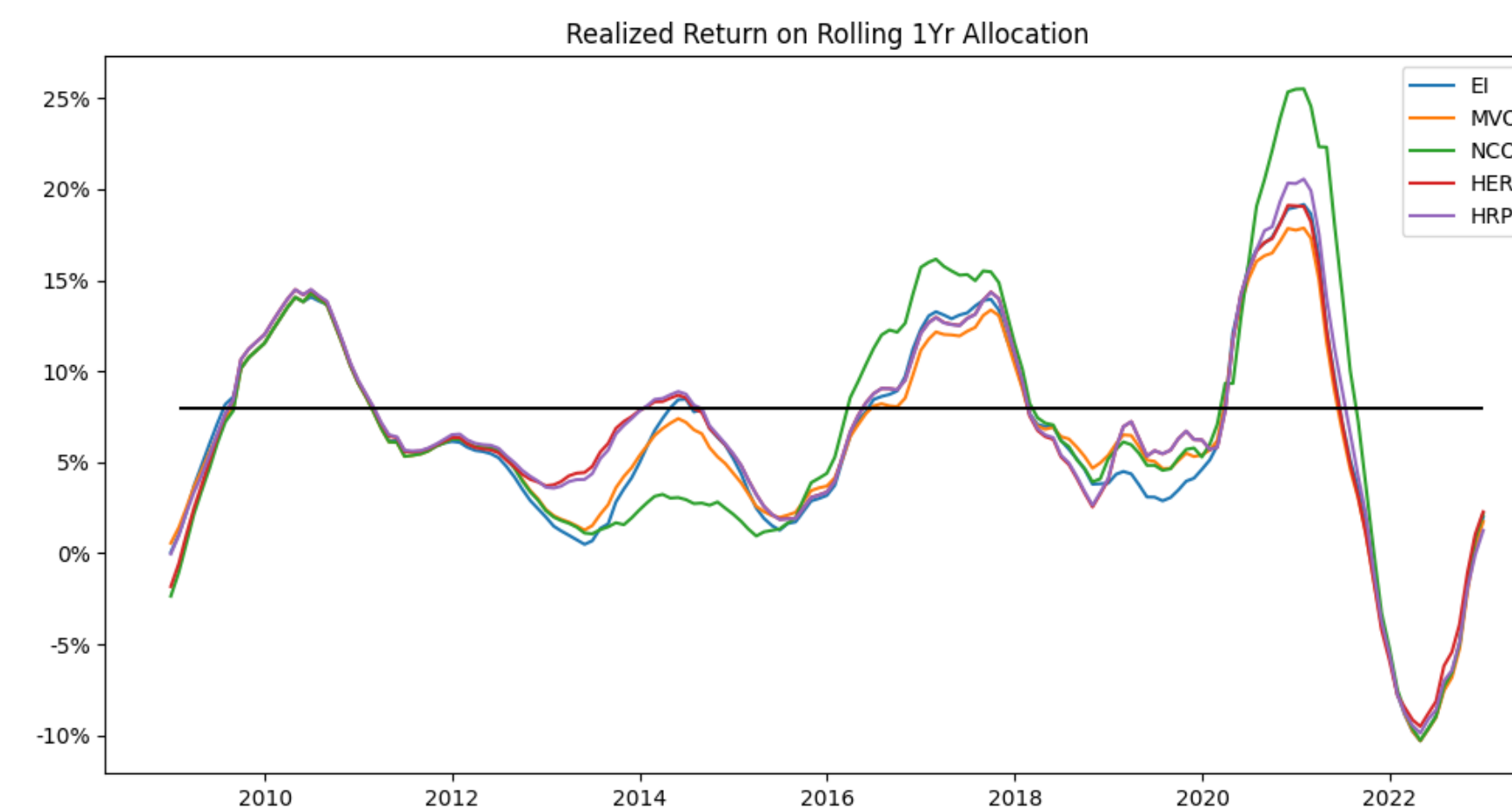
When including return and volatility we get a smooth surface as it is a quadratic problem. As more measures are added, the optimization surface might become irregular and it could potentially lead to multiple points where the gradient descend gets stuck. For example, Up Period Percent is not a smooth surface as the measure can only have certain interval values.

In this example we can see the difference surfaces when only including return and volatility, all smooth surfaces, and a group of 30 performance measures, which some are Smooth and some aren't.



Model Comparisons

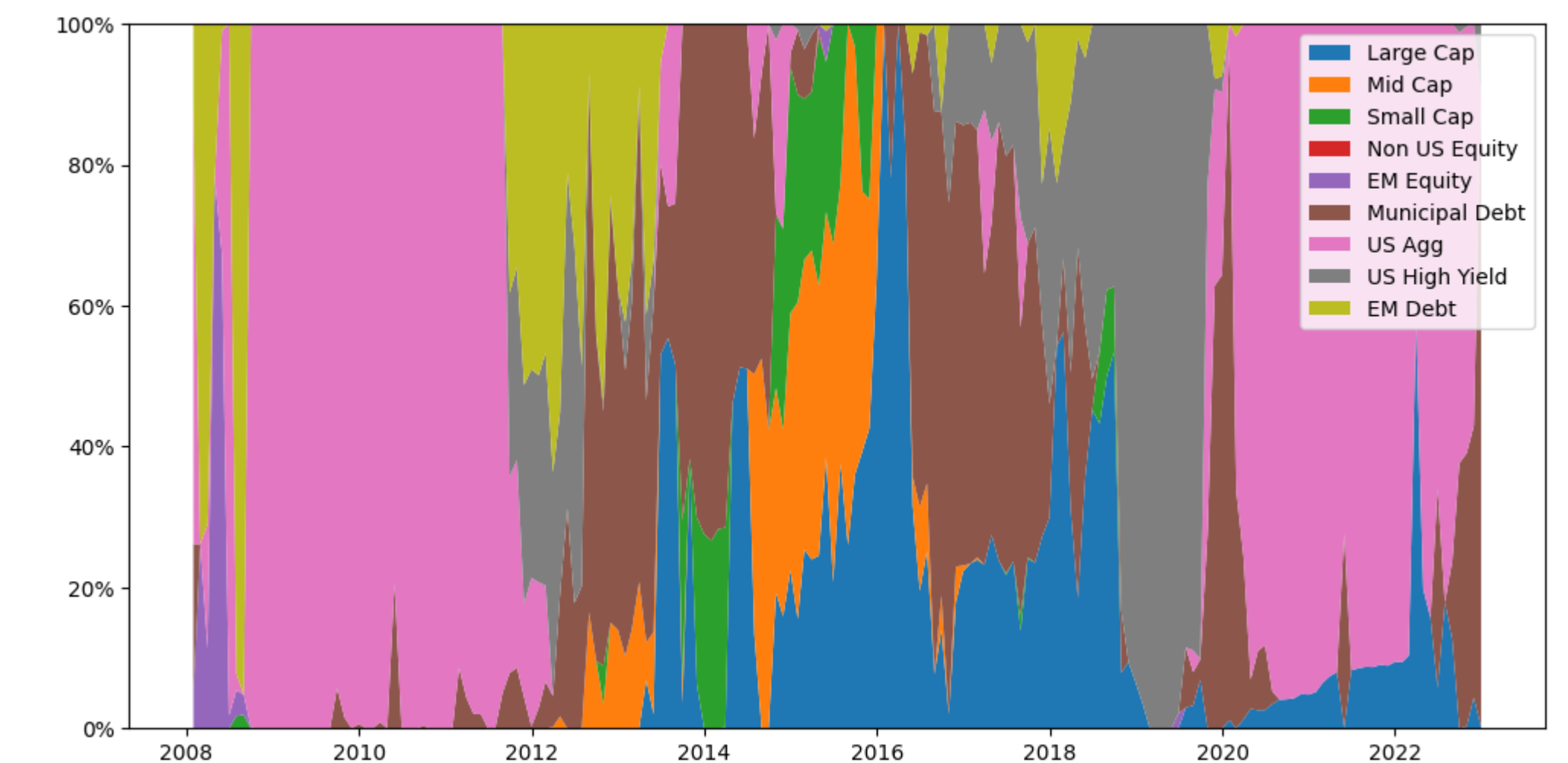
When comparing EI with other single objective models, like MVO, NCO, HRP, and HERC, we see that EI performs in par as the other models. Below we have an example where we aim to minimize volatility (all volatility related measures in the case of EI) where we utilize the historical 5 year performance as the market expectations for the next year through time. We assume that each month we come up with a new allocation will be hold for the next year. All models also include the constraint requiring to obtain a return greater than 8%



	EI		MVO		NCO		HRP		HERC	
	Ret	Vol	Ret	Vol	Ret	Vol	Ret	Vol	Ret	Vol
Q1	1.87%	2.90%	2.34%	2.95%	1.52%	2.86%	2.34%	3.45%	2.35%	3.47%
Median	5.73%	4.77%	5.86%	4.85%	5.30%	5.13%	6.43%	5.34%	6.43%	5.31%
Q3	10.14%	7.22%	9.39%	7.69%	11.13%	9.82%	10.58%	9.09%	10.61%	9.26%

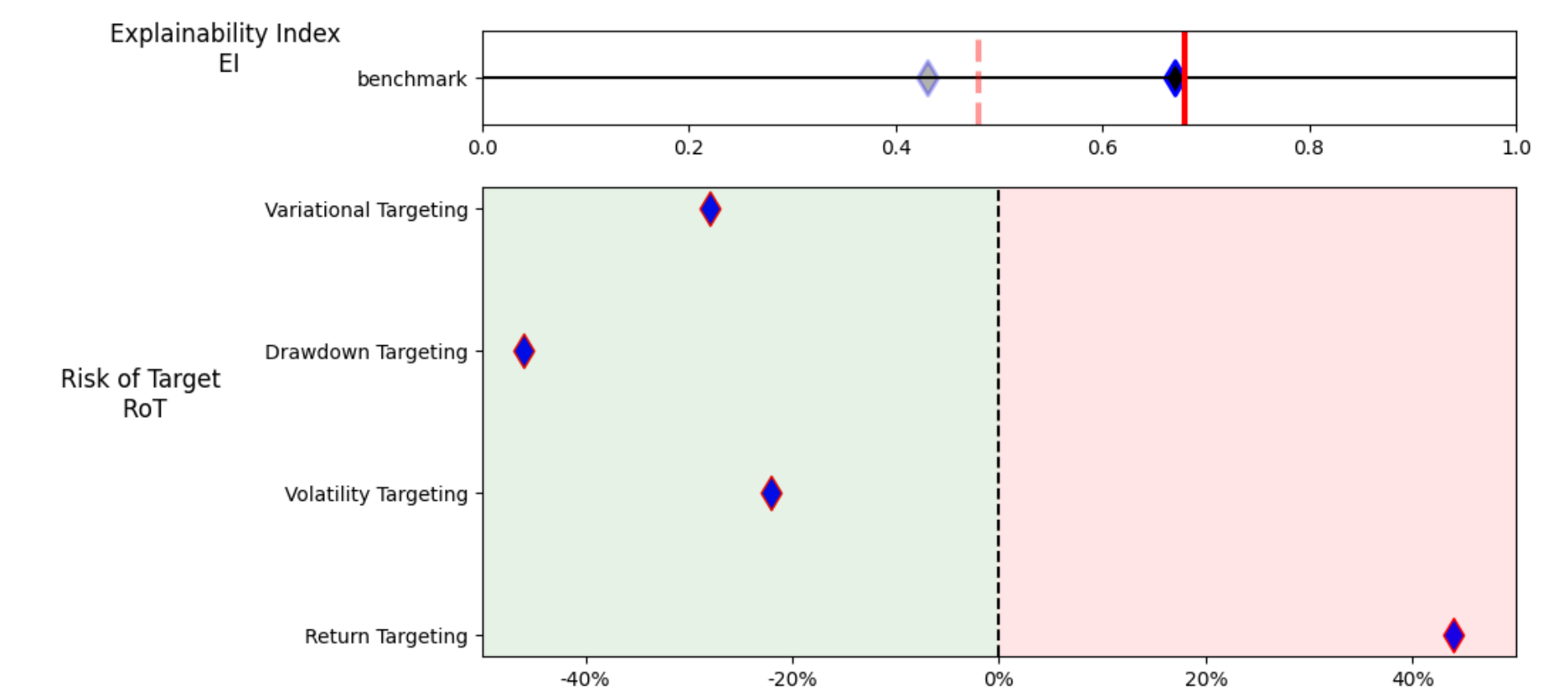
Optimizing a multi-objective allocation

Based on the four default categories (1) and equally weighted, we construct a multi-objective allocation. This allocation will be constructed based on 10 different equity and fixed income indexes. No constraints are included in this example as the only objective is to minimize EI.



	Return	volatility	MDD	Sharpe	Calmar
Q1	1.88%	2.94%	1.44%	0.39	0.51
Median	5.10%	3.95%	2.67%	1.21	2.03
Q3	7.60%	5.55%	4.38%	2.25	5.5

We can see that the allocation ended up being a relatively conservative allocation. We can adjust the weights of the EI categories to adjust to the user risk profile. Below we can compare the risk profile and EI of this allocation vs a 60-40 allocation.



References

- 1) Hirsa, Ali, Rui Ding, and Satyan Malhotra. "Explainability Index (EI): Unifying Framework of Performance Measures and Risk of Target (RoT): Variability from Target EI." *Available at SSRN 4335455* (2023).