

Dynamic Portfolio Rebalance with Tax Optimization and Mandate Constraints

Jared Cohen – Advisors: Ali Hirs, Miao Wang

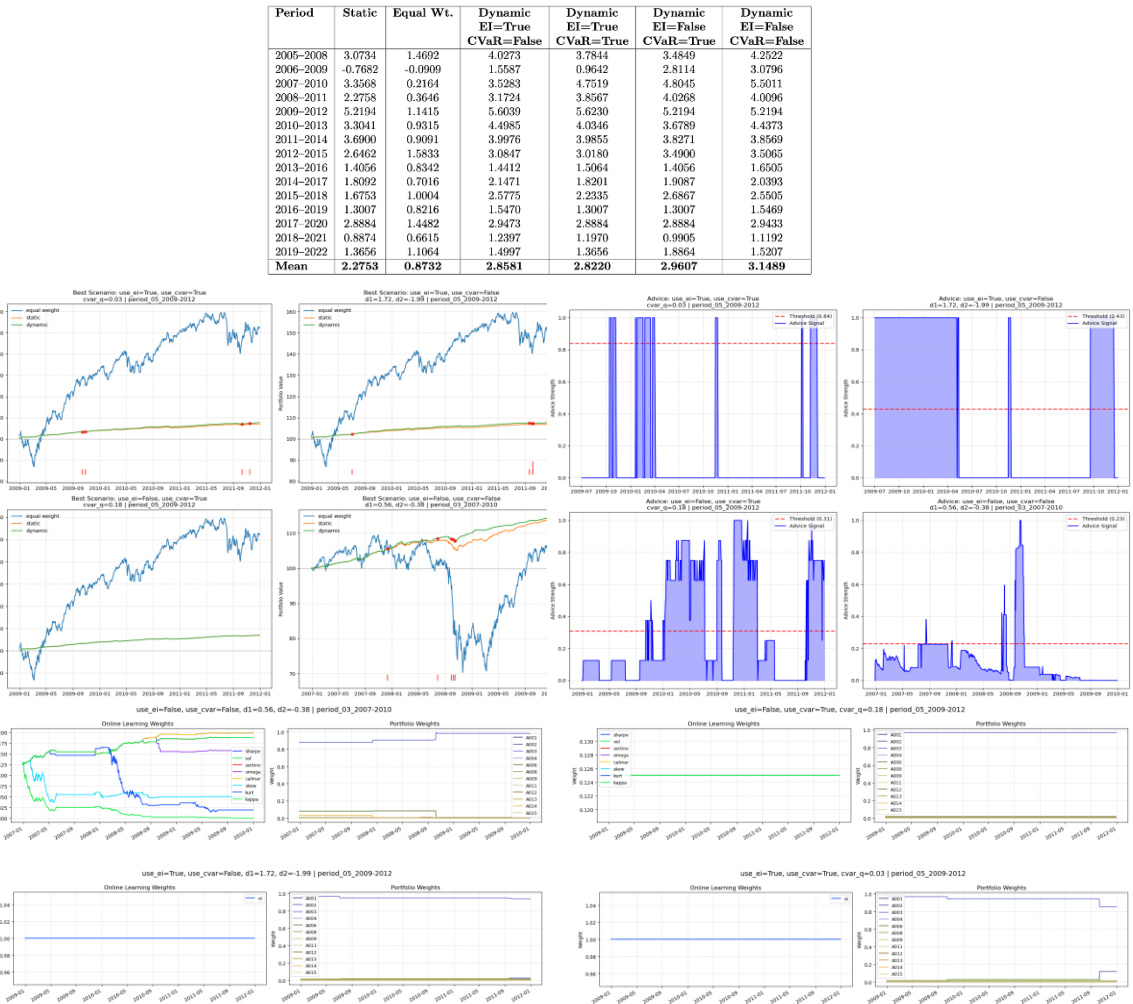


Abstract

This paper introduces a dynamic portfolio rebalancing framework that incorporates real-world constraints—such as transaction costs, taxes, and trading frequency limits—across multiple optimization objectives. Instead of trading continuously, it uses adaptive thresholds and rank-change filtering to trigger rebalancing only when justified. Tested configurations demonstrate that the framework successfully improves portfolio performance compared to static benchmarks while effectively minimizing turnover and tax costs. It also supports equity-share and factor-exposure bands, which constrain each rebalance and can optionally signal when to trade.

Unconstrained Case

Performance Metric	EI=True CVar=False	EI=True CVar=True	EI=False CVar=True	EI=False CVar=False
Total Periods Analyzed	15	15	15	15
Mean Sharpe Ratio	2.8581	2.8220	2.9607	3.1489
Standard Deviation	1.2640	1.4152	1.2350	1.3487
Best Performance	Period 5: 5.6039	Period 5: 5.6230	Period 5: 5.2194	Period 3: 5.5011
Worst Performance	Period 14: 1.2397	Period 2: 0.9642	Period 14: 0.9905	Period 14: 1.1192
Performance Range	4.3642	4.6588	4.2289	4.3819
Consistency Score	0.4417	0.4140	0.4474	0.4258



Methodology

1. Target Allocation & Transaction Costs

Core Model: Computes long-only, fully invested target weights using mean–variance optimization across flexible objectives (e.g., Max Return, Min Volatility, Max Sharpe/Calmar).

Cost Integration: Directly penalizes the optimization objective using linear or quadratic transaction costs based on weight turnover.

2. Multi-Metric Performance Triggers

Unconstrained Case: Tracks 8 historical metrics (Sharpe, Volatility, Sortino, etc.) using adaptive thresholds: **Bollinger Bands** (historical mean/SD cutoffs) or **CVaR** (historical lower-tail averages). Signals are evaluated independently or aggregated into a single score via the **Explainability Index (EI)**.

Constrained Case: Disables EI/ CVaR. Maps the specific optimization objective and each active portfolio constraint to its own trigger, centering thresholds around strict constraint limits.

Structural Bands: Optionally constrains equity share to a target ± 0.05 band (30 / 50 / 70%) and Fama-French 5-factor exposures to ± 0.15 . Each band is a hard constraint inside the MVO on every rebalance, holding the portfolio inside the mandate, and each can optionally also cast a vote in the rebalance decision.

3. Execution Filter & Online Learning Loop

Rank-Change Filter: Blocks trading unless at least one asset's relative rank shift exceeds a minimum threshold, eliminating negligible adjustments.

Adaptive Weights: Executes rebalancing only if the filter passes and total trigger advice exceeds a threshold. An **Online Learning (OL)** loop dynamically scales trigger weights using a frequency-adjusted reward; successful signals are amplified, while poor signals are penalized.

4. Simulation Penalties & Parameter Search

Two-Tier Penalties: Sequential penalties are applied directly to the global simulation objective ($J_{\text{final}} = J_{\text{freq}} + |J_{\text{freq}}|\xi$) to prevent overfitting to high-turnover strategies.

Frequency: Linearly penalizes strategies exceeding 5 trades per 3-year period.

A capped or exponential penalty makes rebalances free once it saturates; linear has a monotone marginal cost at every frequency.

Tax Module: Tracks asset cost-bases (\$10k), nets capital gains, applies U.S. tax rates (40% ST / 20% LT), and penalizes the portfolio by the total tax liability relative to final value.

Optimization Initialization: Uses a global **interpolation grid search** to find stable initial parameters, paired with a **warm-start** mechanism across rolling windows to avoid poor initialization and maintain temporal continuity.

Results

Outperformed Benchmarks: In sample, all four unconstrained dynamic setups beat static and equal-weight strategies. The constrained dynamic strategy improves the mean Sharpe ratio from 0.87 for equal-weight and 1.59 for the static to 2.07.

Crisis Protection: Dynamic rules protected capital during the 2008 financial crisis, turning a static Sharpe ratio (-0.77) into positive returns (2.13) in the constrained setup.

Controlled Trading: The framework successfully minimized unnecessary turnover, averaging just 2.8 to 4.1 rebalances per 3-year period across setups for unconstrained and 1.7 for constrained.

Low Tax Impact: Tax drag remained highly controlled, keeping the average portfolio tax penalty at or below 1.0% across the four unconstrained configurations.

Mandates Do Not Break It: Adding equity-share or factor-exposure bands, all 8 of 8 banded arms beat a static portfolio carrying the identical bands — mean +0.098 Sharpe, two significant at $p < 0.05$. The mandate lowers absolute Sharpe; rebalancing inside it still adds value.

Stable Initialization: Interpolation search with warm-starts left mean objective performance largely unchanged but reduced its volatility.

Future Works

Granular Taxes: Track multi-layer cost-basis for partial sales; scale portfolio to \$100k+.

Hybrid Search: Combine Nelder-Mead with Bayesian or genetic algorithms to avoid local optima.

Adaptive Penalties: Scale frequency/tax penalties dynamically based on market volatility.

Regime Detection: Condition parameter choice on a detected market regime instead of assuming the next period resembles this one.

Constrained Case

Performance Metric	Value
Total Periods Analyzed	15
Mean Sharpe Ratio	2.0704
Standard Deviation	0.8350
Best Performance	Period 7: 3.4460
Worst Performance	Period 14: 0.8874
Performance Range	2.5586
Consistency Score	0.5450

Period	Static	Dynamic	Equal Weight
2005–2008	2.7815	3.0972	1.4692
2006–2009	-0.7683	2.1334	-0.0909
2007–2010	0.9135	1.3288	0.2164
2008–2011	1.5602	2.2713	0.3656
2009–2012	2.6586	2.8148	1.1415
2010–2013	1.6869	3.3499	0.9315
2011–2014	3.4460	3.4460	0.9091
2012–2015	2.6464	2.9874	1.5833
2013–2016	1.3538	1.3538	0.8342
2014–2017	1.8083	1.8083	0.7016
2015–2018	1.5079	1.6253	1.0004
2016–2019	0.6951	1.2703	0.8216
2017–2020	1.3163	1.3163	1.4482
2018–2021	0.8874	0.8874	0.6615
2019–2022	1.3656	1.3656	1.1064
Mean	1.5906	2.0704	0.8732

Mandate	Dyn	Stat	Δ
none	1.716	1.584	+0.132
eq 30%	1.332	1.150	+0.181
eq 50%	1.026	0.889	+0.137
eq 70%	0.813	0.736	+0.078
factor	1.777	1.613	+0.164
eq30+f	1.364	1.316	+0.048
eq50+f	1.110	1.080	+0.030
eq70+f	0.934	0.922	+0.012

eq = equity-share band, target ± 0.05 . f = FF5 factor bands, ± 0.15 .

